1.2 Relations

1.2.1 Definition

Let A and B be two non-empty sets, then every subset of $A \times B$ defines a relation from A to B and every relation from A to B is a subset of $A \times B$.

Let $R \subseteq A \times B$ and $(a, b) \in R$. Then we say that a is related to b by the relation R and write it as $a R b$. If $(a,b) \in R$, we write it as $a R b$.

Example: Let $A = \{1, 2, 5, 8, 9\}$, $B = \{1, 3\}$ we set a relation from A to B as: $a\ R\ b$ iff $a \le b$; $a \in A, b \in B$. Then $R = \{(1, 1)\}, (1, 3), (2, 3)\} \subset A \times B$

(1) **Total number of relations :** Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So, total number of subset of $A \times B$ is 2^{mn}. Since each subset of $A \times B$ defines relation from A to B, so total number of relations from A to B is 2^{mn}. Among these 2^{mn} relations the void relation ϕ and the universal relation $A \times B$ are trivial relations from A to B.

(2) **Domain and range of a relation :** Let R be a relation from a set A to a set B. Then the set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R, while the set of all second components or coordinates of the ordered pairs in R is called the range of R.

Thus, Dom $(R) = \{a : (a, b) \in R\}$ and Range $(R) = \{b : (a, b) \in R\}$.

It is evident from the definition that the domain of a relation from A to B is a subset of A and its range is a subset of B.

(3) **Relation on a set :** Let A be a non-void set. Then, a relation from A to itself *i.e.* a subset of $A \times A$ is called a relation on set A.

Example: 1 Let $A = \{1, 2, 3\}$. The total number of distinct relations that can be defined over A is

(a) 2^9 (b) 6 (c) 8 (d) None of these

Solution: (a) $n(A \times A) = n(A).n(A) = 3^2 = 9$

So, the total number of subsets of $A \times A$ is 2^9 and a subset of $A \times A$ is a relation over the set A.

Example: 2 Let $X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 3, 5, 7, 9\}$. Which of the following is/are relations from X to Y

(a) $R_1 = \{(x, y) | \ y = 2 + x, x \in X, y \in Y\}$ (b) $R_2 = \{(1,1),(2,1),(3,3),(4,3),(5,5)\}$

(c) $R_3 = \{(1,1),(1,3)(3,5),(3,7),(5,7)\}$ (d) $R_4 = \{(1,3),(2,5),(2,4),(7,9)\}$

Solution: (a,b,c) R_4 is not a relation from X to Y, because $(7, 9) \in R_4$ but $(7, 9) \notin X \times Y$.

Example: 3 Given two finite sets A and B such that $n(A) = 2$, $n(B) = 3$. Then total number of relations from A to B is

(a) 4 (b) 8 (c) 64 (d) None of these

Solution: (c) Here $n(A\times B)$ = 2 × 3 = 6

Since every subset of $A \times B$ defines a relation from A to B, number of relation from A to B is equal to number of subsets of $A\times B=2^6=64$, which is given in (c).

Example: 4 The relation R defined on the set of natural numbers as {(a, b) : a differs from b by 3}, is given by

(a) {(1, 4, (2, 5), (3, 6),.....} (b) {(4, 1), (5, 2), (6, 3),.....} (c){(1, 3), (2, 6), (3, 9),..}

Solution: (b) $R=\{(a,b):a,b\in N, a-b=3\}=\{((n+3),n):n\in N\}=\{(4,1),(5,2),(6,3).....\}$

1.2.2 Inverse Relation

Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1}=\{(b,a):(a,b)\in R\}$

Clearly (a, b) ∈ R ⇔ (b, a) ∈ R^{-1}. Also, Dom (R) = Range (R^{-1}) and Range (R) = Dom (R^{-1})

Example : Let A = {a, b, c}, B = {1, 2, 3} and R = {(a, 1), (a, 3), (b, 3), (c, 3)}.

Then, (i) R^{-1} = {(1, a), (3, a), (3, b), (3, c)}

(ii) Dom (R) = {a, b, c} = Range (R^{-1})

(iii) Range (R) = {1, 3} = Dom (R^{-1})

Example: 5 Let A = {1, 2, 3}, B = {1, 3, 5}. A relation $R: A\to B$ is defined by R = {(1, 3), (1, 5), (2, 1)}. Then R^{-1} is defined by

(a) {(1,2), (3,1), (1,3), (1,5)} (b) {(1, 2), (3, 1), (2, 1)} (c) {(1, 2), (5, 1), (3, 1)}(d)

Solution: (c) $(x,y)\in R\Leftrightarrow(y,x)\in R^{-1}$, $\therefore$ $R^{-1}=\{(3,1),(5,1),(1,2)\}$.

Example: 6 The relation R is defined on the set of natural numbers as {(a, b) : a = 2b}. Then R^{-1} is given by

(a) {(2, 1), (4, 2), (6, 3).....} (b) {(1, 2), (2, 4), (3, 6)....}(c) R^{-1} is not defined (d)

Solution: (b) R = {(2, 1), (4, 2), (6, 3),......} So, R^{-1} = {(1, 2), (2, 4), (3, 6),.....}.

1.2.3 Types of Relations

(1) **Reflexive relation :** A relation R on a set A is said to be reflexive if every element of A is related to itself.

Thus, R is reflexive ⇔ (a, a) ∈ R for all a ∈ A.

A relation R on a set A is not reflexive if there exists an element a ∈ A such that (a, a) ∉ R.

Example: Let A = {1, 2, 3} and R = {(1, 1); (1, 3)}

Then R is not reflexive since $3\in A$ but (3, 3) ∉ R

Note : ❑ The identity relation on a non-void set A is always reflexive relation on A. However, a reflexive relation on A is not necessarily the identity relation on A.

❑ The universal relation on a non-void set A is reflexive.

(2) **Symmetric relation :** A relation R on a set A is said to be a symmetric relation *iff*

(a, b) ∈ R ⇒ (b, a) ∈ R for all a, b ∈ A

i.e. $aRb \Rightarrow bRa$ for all $a, b \in A$.

it should be noted that R is symmetric iff $R^{-1} = R$

Note : ❑ The identity and the universal relations on a non-void set are symmetric relations.

❑ A relation R on a set A is not a symmetric relation if there are at least two elements $a, b \in A$ such that $(a, b) \in R$ but $(b, a) \notin R$.

❑ A reflexive relation on a set A is not necessarily symmetric.

(3) **Anti-symmetric relation :** Let A be any set. A relation R on set A is said to be an anti-symmetric relation *iff* $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b$ for all $a, b \in A$.

Thus, if $a \neq b$ then a may be related to b or b may be related to a, but never both.

Example: Let N be the set of natural numbers. A relation $R \subseteq N \times N$ is defined by xRy iff x divides y(*i.e.*, x/y).

Then $x\,R\,y,\ y\,R\,x \Rightarrow x$ divides y, y divides $x \Rightarrow x = y$

Note : ❑ The identity relation on a set A is an anti-symmetric relation.

❑ The universal relation on a set A containing at least two elements is not anti-symmetric, because if $a \neq b$ are in A, then a is related to b and b is related to a under the universal relation will imply that $a = b$ but $a \neq b$.

❑ The set $\{(a, a) : a \in A\} = D$ is called the diagonal line of $A \times A$. Then "the relation R in A is antisymmetric iff $R \cap R^{-1} \subseteq D$".

(4) **Transitive relation :** Let A be any set. A relation R on set A is said to be a transitive relation *iff*

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$ *i.e.*, aRb and $bRc \Rightarrow aRc$ for all $a, b, c \in A$.

In other words, if a is related to b, b is related to c, then a is related to c.

Transitivity fails only when there exists a, b, c such that $a\,R\,b, b\,R\,c$ but $a\,R\,c$.

Example: Consider the set $A = \{1, 2, 3\}$ and the relations

$R_1 = \{(1, 2), (1, 3)\}$; $R_2 = \{(1, 2)\}$; $R_3 = \{(1, 1)\}$; $R_4 = \{(1, 2), (2, 1), (1, 1)\}$

Then R_1, R_2, R_3 are transitive while R_4 is not transitive since in $R_4, (2, 1) \in R_4; (1, 2) \in R_4$ but $(2, 2) \notin R_4$.

Note : ❑ The identity and the universal relations on a non-void sets are transitive.

❑ The relation 'is congruent to' on the set T of all triangles in a plane is a transitive relation.

(5) **Identity relation :** Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A.

In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only. Every identity relation will be reflexive, symmetric and transitive.

Example: On the set = {1, 2, 3}, R = {(1, 1), (2, 2), (3, 3)} is the identity relation on A .

Note : ❑ It is interesting to note that every identity relation is reflexive but every reflexive relation need not be an identity relation.

Also, identity relation is reflexive, symmetric and transitive.

(6) **Equivalence relation :** A relation R on a set A is said to be an equivalence relation on A *iff*

(i) It is reflexive *i.e.* $(a, a) \in R$ for all $a \in A$

(ii) It is symmetric *i.e.* $(a, b) \in R \Rightarrow (b, a) \in R$, for all $a, b \in A$

(iii) It is transitive *i.e.* $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$.

Note : ❑ **Congruence modulo (m) :** Let m be an arbitrary but fixed integer. Two integers a and b are said to be congruence modulo m if $a-b$ is divisible by m and we write $a \equiv b$ (mod m).

Thus $a \equiv b$ (mod m) $\Leftrightarrow a-b$ is divisible by m. For example, $18 \equiv 3$ (mod 5) because 18 – 3 = 15 which is divisible by 5. Similarly, $3 \equiv 13$ (mod 2) because 3 – 13 = –10 which is divisible by 2. But 25 ≠ 2 (mod 4) because 4 is not a divisor of 25 – 3 = 22.

The relation "Congruence modulo m" is an equivalence relation.

Important Tips

- *If R and S are two equivalence relations on a set A, then $R \cap S$ is also an equivalence relation on A.*
- *The union of two equivalence relations on a set is not necessarily an equivalence relation on the set.*
- *The inverse of an equivalence relation is an equivalence relation.*

1.2.4 Equivalence Classes of an Equivalence Relation

Let R be equivalence relation in $A(\neq \phi)$. Let $a \in A$. Then the equivalence class of a, denoted by $[a]$ or $\{\overline{a}\}$ is defined as the set of all those points of A which are related to a under the relation R. Thus $[a] = \{x \in A : x R a\}$.

It is easy to see that

(1) $b \in [a] \Rightarrow a \in [b]$ (2) $b \in [a] \Rightarrow [a] = [b]$ (3) Two equivalence classes are either disjoint or identical.

As an example we consider a very important equivalence relation $x \equiv y (\text{mod } n)$ iff n divides $(x-y), n$ is a fixed positive integer. Consider $n = 5$. Then

$[0] = \{x : x \equiv 0 \text{ (mod 5)}\} = \{5p : p \in Z\} = \{0, \pm 5, \pm 10, \pm 15, ...\}$

$[1] = \{x : x \equiv 1(\text{mod } 5)\} = \{x : x - 1 = 5k, k \in Z\} = \{5k + 1 : k \in Z\} = \{1, 6, 11, ..., -4, -9, ...\}$.

One can easily see that there are only 5 distinct equivalence classes *viz*. [0], [1], [2], [3] and [4], when $n = 5$.

Example: 7 Given the relation $R = \{(1, 2), (2, 3)\}$ on the set $A = \{1, 2, 3\}$, the minimum number of ordered pairs which when added to R make it an equivalence relation is

(a) 5 (b) 6 (c) 7 (d) 8

Solution: (c) R is reflexive if it contains (1, 1), (2, 2), (3, 3)

$\because (1,2) \in R, (2,3) \in R$

$\therefore$ R is symmetric if $(2, 1), (3, 2) \in R$. Now, $R = \{(1,1), (2,2), (3,3), (2,1), (3,2), (2,3), (1,2)\}$

R will be transitive if $(3, 1); (1, 3) \in R$. Thus, R becomes an equivalence relation by adding (1, 1) (2, 2) (3, 3) (2, 1) (3,2) (1, 3) (3, 1). Hence, the total number of ordered pairs is 7.

Example: 8 The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on set $A = \{1, 2, 3\}$ is

(a) Reflexive but not symmetric (b) Reflexive but not transitive

(c) Symmetric and Transitive (d) Neither symmetric nor transitive

Solution: (a) Since $(1, 1); (2, 2); (3, 3) \in R$ therefore R is reflexive. $(1, 2) \in R$ but $(2, 1) \notin R$, therefore R is not symmetric. It can be easily seen that R is transitive.

Example: 9 Let R be the relation on the set R of all real numbers defined by $a R b$ iff $|a - b| \le 1$. Then R is

(a) Reflexive and Symmetric (b) Symmetric only (c) Transitive only (d)

Solution: (a) $|a - a| = 0 < 1 \quad \therefore a R a \, \forall a \in R$

$\therefore$ R is reflexive, Again $a R b \Rightarrow |a - b| \le 1 \Rightarrow |b - a| \le 1 \Rightarrow bRa$

$\therefore$ R is symmetric, Again $1 R \frac{1}{2}$ and $\frac{1}{2} R 1$ but $\frac{1}{2} \neq 1$

$\therefore$ R is not anti-symmetric

Further, $1 R 2$ and $2 R 3$ but $1 \not{R} 3$

$[\because |1 - 3| = 2 > 1]$

$\therefore$ R is not transitive.

Example: 10 The relation "less than" in the set of natural numbers is **[UPSEAT 1994, 98; AMU 1999]**

(a) Only symmetric (b) Only transitive (c) Only reflexive (d) Equivalence relation

Solution: (b) Since $x < y, y < z \Rightarrow x < z \; \forall \, x, y, z \in N$

$\therefore \; x R y, y R z \Rightarrow x R z$, $\therefore$ Relation is transitive , $\therefore \; x < y$ does not give $y < x$, $\therefore$ Relation is not symmetric.

Since $x < x$ does not hold, hence relation is not reflexive.

Example: 11 With reference to a universal set, the inclusion of a subset in another, is relation, which is

(a) Symmetric only (b) Equivalence relation (c) Reflexive only (d) None of these

Solution: (d) Since $A \subseteq A$ $\therefore$ relation '$\subseteq$' is reflexive.

Since $A \subseteq B$, $B \subseteq C \Rightarrow A \subseteq C$

$\therefore$ relation '$\subseteq$' is transitive.

But $A \subseteq B$, $\not\Rightarrow B \subseteq A$, $\therefore$ Relation is not symmetric.

Example: 12 Let $A = \{2, 4, 6, 8\}$. A relation R on A is defined by $R = \{(2,4),(4,2),(4,6),(6,4)\}$. Then R is

(a) Anti-symmetric (b) Reflexive (c) Symmetric (d) Transitive

Solution: (c) Given A = {2, 4, 6, 8}

R = {(2, 4)(4, 2) (4, 6) (6, 4)}

(a, b) $\in R \Rightarrow$ (b, a) $\in R$ and also $R^{-1} = R$. Hence R is symmetric.

Example: 13 Let $P = \{(x,y) \mid x^2 + y^2 = 1, x, y \in R\}$. Then P is

(a) Reflexive (b) Symmetric (c) Transitive (d) Anti-symmetric

Solution: (b) Obviously, the relation is not reflexive and transitive but it is symmetric, because $x^2 + y^2 = 1 \Rightarrow y^2 + x^2 = 1$.

Example: 14 Let R be a relation on the set N of natural numbers defined by $nRm \Leftrightarrow n$ is a factor of m (*i.e.*, $n|m$). Then R is

(a) Reflexive and symmetric (b) Transitive and symmetric

(c) Equivalence (d) Reflexive, transitive but not symmetric

Solution: (d) Since $n \mid n$ for all $n \in N$, therefore R is reflexive. Since $2 \nmid 6$ but $6 \mid 2$, therefore R is not symmetric.

Let $n\ R\ m$ and $m\ R\ p \Rightarrow n|m$ and $m|p \Rightarrow n|p \Rightarrow nRp$. So R is transitive.

Example: 15 Let R be an equivalence relation on a finite set A having n elements. Then the number of ordered pairs in R is

(a) Less than n (b) Greater than or equal to n (c) Less than or equal to n(d)

Solution: (b) Since R is an equivalence relation on set A, therefore (a, a) $\in R$ for all $a \in A$. Hence, R has at least n ordered pairs.

Example: 16 Let N denote the set of all natural numbers and R be the relation on $N \times N$ defined by (a, b) R (c, d) if $ad(b+c) = bc(a+d)$, then R is **[Roorkee 1995]**

(a) Symmetric only (b) Reflexive only (c) Transitive only (d) An equivalence relation

Solution: (d) For (a, b), (c, d) $\in N \times N$

$(a,b)R(c,d) \Rightarrow ad(b+c) = bc(a+d)$

Reflexive: Since $ab(b+a) = ba(a+b) \forall ab \in N$,

$\therefore$ $(a,b)R(a,b)$, $\therefore$ R is reflexive.

Symmetric: For $(a,b),(c,d) \in N \times N$, let $(a,b)R(c,d)$

$\therefore$ $ad(b+c) = bc(a+d) \Rightarrow bc(a+d) = ad(b+c) \Rightarrow cb(d+a) = da(c+b) \Rightarrow (c,d)R(a,b)$

$\therefore$ R is symmetric

Transitive: For $(a,b),(c,d),(e,f) \in N \times N$, Let $(a,b)R(c,d),(c,d)R(e,f)$

$\therefore$ $ad(b+c) = bc(a+d)$, $cf(d+e) = de(c+f)$

$\Rightarrow adb + adc = bca + bcd$ .....(i) and $cfd + cfe = dec + def$ ....(ii)

(i) × ef + (ii) × ab gives, $adbef + adcef + cfdab + cfeab = bcaef + bcdef + decab + defab$

$\Rightarrow adcf(b+e) = bcde(a+f) \Rightarrow af(b+e) = be(a+f) \Rightarrow (a,b)R(e,f)$. $\therefore$ R is transitive. Hence R is an equivalence relation.

Example: 17 For real numbers x and y, we write $x\ Ry \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is

(a) Reflexive (b) Symmetric (c) Transitive (d) None of these

Solution: (a) For any $x \in R$, we have $x - x + \sqrt{2} = \sqrt{2}$ an irrational number.

$\Rightarrow xRx$ for all x. So, R is reflexive.

R is not symmetric, because $\sqrt{2} R\not{1}$ but $1 R \sqrt{2}$, R is not transitive also because $\sqrt{2}\ R\not{1}$ and $1R2\sqrt{2}$ but $\sqrt{2}\ R\ 2\sqrt{2}$.

Example: 18 Let X be a family of sets and R be a relation on X defined by 'A is disjoint from B'. Then R is

(a) Reflexive (b) Symmetric (c) Anti-symmetric (d) Transitive

Solution: (b) Clearly, the relation is symmetric but it is neither reflexive nor transitive.

Example: 19 Let R and S be two non-void relations on a set A. Which of the following statements is false

(a) R and S are transitive $\Rightarrow R \cup S$ is transitive (b) R and S are transitive $\Rightarrow R \cap S$ is transitive

(c) R and S are symmetric $\Rightarrow R \cup S$ is symmetric (d) R and S are reflexive $\Rightarrow R \cap S$ is reflexive

Solution: (a) Let $A = \{1, 2, 3\}$ and R = {(1, 1), (1, 2)}, S = {(2, 2) (2, 3)} be transitive relations on A.

Then $R \cup S$ = {(1, 1); (1, 2); (2, 2); (2, 3)}

Obviously, $R \cup S$ is not transitive. Since (1, 2) $\in R \cup S$ and $(2,3) \in R \cup S$ but (1, 3) $\notin R \cup S$.

Example: 20 The solution set of $8x \equiv 6 (\text{mod } 14), x \in Z$, are

(a) $[8] \cup [6]$ (b) $[8] \cup [14]$ (c) $[6] \cup [13]$ (d) $[8] \cup [6] \cup [13]$

Solution: (c) $8x - 6 = 14P (P \in Z) \Rightarrow x = \frac{1}{8}[14P + 6], x \in Z$

$\Rightarrow x = \frac{1}{4}(7P + 3) \Rightarrow x$ = 6, 13, 20, 27, 34, 41, 48,.......

$\therefore$ Solution set = {6, 20, 34, 48,.....} $\cup$ {13, 27, 41, ...} = [6] $\cup$ [13].

Where [6], [13] are equivalence classes of 6 and 13 respectively.

1.2.5 Composition of Relations

Let R and S be two relations from sets A to B and B to C respectively. Then we can define a relation SoR from A to C such that $(a, c) \in SoR \Leftrightarrow \exists\ b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.

This relation is called the composition of R and S.

For example, if A = {1, 2, 3}, B = {a, b, c, d}, C = {p, q, r, s} be three sets such that R = {(1, a), (2, c), (1, c), (2, d)} is a relation from A to B and S = {(a, s), (b, q), (c, r)} is a relation from B to C. Then SoR is a relation from A to C given by SoR = {(1, s) (2, r) (1, r)}

In this case RoS does not exist.

In general $RoS \neq SoR$. Also $(SoR)^{-1} = R^{-1}oS^{-1}$.

Example: 21 If R is a relation from a set A to a set B and S is a relation from B to a set C, then the relation SoR

(a) Is from A to C (b) Is from C to A (c) Does not exist (d) None of these

Solution: (a) It is obvious.

Example: 22 If $R \subset A \times B$ and $S \subset B \times C$ be two relations, then $(SoR)^{-1} =$

(a) $S^{-1}oR^{-1}$ (b) $R^{-1}oS^{-1}$ (c) SoR (d) RoS

Solution: (b) It is obvious.

Example: 23 If R be a relation < from A = {1,2, 3, 4} to B = {1, 3, 5} *i.e.,* $(a, b) \in R \Leftrightarrow a < b$, then RoR^{-1} is

(a) {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)}

(b) {(3, 1) (5, 1), (3, 2), (5, 2), (5, 3), (5, 4)}

(c) {(3, 3), (3, 5), (5, 3), (5, 5)}

(d) {(3, 3) (3, 4), (4, 5)}

Solution: (c) We have, R = {(1, 3); (1, 5); (2, 3); (2, 5); (3, 5); (4, 5)}

R^{-1} = {(3, 1), (5, 1), (3, 2), (5, 2); (5, 3); (5, 4)}

Hence RoR^{-1} = {(3, 3); (3, 5); (5, 3); (5, 5)}

Example: 24 Let a relation R be defined by R = {(4, 5); (1, 4); (4, 6); (7, 6); (3, 7)} then $R^{-1}oR$ is

(a) {(1, 1), (4, 4), (4, 7), (7, 4), (7, 7), (3, 3)} (b) {(1, 1), (4, 4), (7, 7), (3, 3)}

(c) {(1, 5), (1, 6), (3, 6)} (d) None of these

Solution: (a) We first find R^{-1}, we have $R^{-1} = \{(5,4);(4,1);(6,4);(6,7);(7,3)\}$ we now obtain the elements of $R^{-1}oR$ we first pick the element of R and then of R^{-1}. Since $(4,5) \in R$ and $(5,4) \in R^{-1}$, we have $(4,4) \in R^{-1}oR$

Similarly, $(1,4) \in R, (4,1) \in R^{-1} \Rightarrow (1,1) \in R^{-1}oR$

$(4,6) \in R, (6,4) \in R^{-1} \Rightarrow (4,4) \in R^{-1}oR,$ $(4,6) \in R, (6,7) \in R^{-1} \Rightarrow (4,7) \in R^{-1}oR$

$(7,6) \in R, (6,4) \in R^{-1} \Rightarrow (7,4) \in R^{-1}oR,$ $(7,6) \in R, (6,7) \in R^{-1} \Rightarrow (7,7) \in R^{-1}oR$

$(3,7) \in R, (7,3) \in R^{-1} \Rightarrow (3,3) \in R^{-1}oR,$

Hence $R^{-1}oR$ = {(1, 1); (4, 4); (4, 7); (7, 4), (7, 7); (3, 3)}.

1.2.6 Axiomatic Definitions of the Set of Natural Numbers (Peano's Axioms)

The set N of natural numbers (N = {1, 2, 3, 4......}) is a set satisfying the following axioms (known as peano's axioms)

(1) N is not empty.

(2) There exist an injective (one-one) map $S : N \rightarrow N$ given by $S(n) = n^+$, where n^+ is the immediate successor of n in N *i.e.,* $n + 1 = n^+$.

(3) The successor mapping S is not surjective (onto).

(4) If $M \subseteq N$ such that,

(i) M contains an element which is not the successor of any element in N, and

(ii) $m \in M \Rightarrow m^+ \in M$, then $M = N$

This is called the axiom of induction. We denote the unique element which is not the successor of any element is 1. Also, we get $1^+ = 2, 2^+ = 3$.

Note : ❑ Addition in N is defined as,

$$n + 1 = n^+$$

$$n + m^+ = (n + m)^+$$

❑ Multiplication in N is defined by,

$$n.1 = n$$

$$n.m^+ = n.m + n$$